# BRIDGE: THE CUTTING EDGE

## APPLICATIONS OF INFORMATION SCIENCE, GAME THEORY AND COMPUTERS TO THE GAME OF BRIDGE

Dave Caprera

FOREWORD BY KIT WOOLSEY

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# DEDICATION

To all the juniors in the USBF Junior Program who will benefit from your purchase of this book.

# INTRODUCTION

The game of bridge has been at close to a standstill for at least 50 years while the world of language, computers and mathematics has experienced so many changes. The bridge techniques and methods your grandmother taught you are probably still the same techniques and methods you use today. The way that bridge problems are approached and analyzed is the same basic use of language, probability and combinatorics that were used a half century ago.

The purpose of this book is to look at the application of modern technology to the game of bridge. In particular, we will focus on information science, game theory and mixed strategy solutions, and computers including artificial intelligence. The book will be divided into these three sections.

You do not have to be an expert bridge player to understand the concepts in this book. And that is the point. Most bridge experts are not familiar with these concepts. Nor do you need to have more than a high school level education in mathematics, although some understanding of probability will be helpful. On the other hand, the book is not intended, by itself, to improve your bridge game to any great degree. Hopefully, though, it will open your eyes to a way of thinking about bridge in the twenty-first century and, in doing so, allow you to make yourself a better player.

In 1975, I graduated from Princeton University with a degree in what today would be called "operations research." It is also where I learned to play bridge. My senior thesis was "A Heuristic Algorithm for Solving the Traveling Salesman Problem." It was published in the journal, *Transportation Science*. (Vol.10, pp. 361-373 (1976)). I couldn't have known it at the time, but it was an application of artificial intelligence before the term was in common use. It involved creating a network which was searched, reduced and replicated. Possible solutions were randomly generated, scored and modified, all on an IBM 360 computer where punch cards were used to provide the input. At Princeton, I was also fortunate to have studied game theory and nonlinear programming under the tutelage of A.W. Tucker and Harold Kuhn. Professor Tucker was the thesis advisor to James Nash, inventor of the “Nash Equilibrium.” These tools remained dormant within me for more than 40 years as I pursued a career as a tax lawyer (not really dormant, as I believe an education in problem solving was very valuable to me in my practice of law). I am now retired and have returned to my roots. Needless to say, so much has changed that I find myself almost starting from scratch. But given bridge’s failure to keep up, I am able to apply my basic understanding of the topic with some additional self-teaching and look at bridge in a way not found in the bridge literature.

# FOREWORD

By Kit Woolsey*

When Dave asked me to write the foreword for his book, I was reluctant. I told him, "I wasn't good at those things." He persisted. I agree with Dave's general assessment that this book will not, in and of itself, make you a better bridge player. But I have always been interested in the concepts that Dave addresses and I concur that this was a book that needed to be written.

This book is not for everyone. Ironically, if you had a background in math and statistics, you may enjoy reading this book even if you don't know how to play bridge (or at least play well.) On the other hand, if you are not analytically inclined and think bridge is just a good "party game," this book may not be for you.

Information theory and bidding efficiency go hand in hand. It is at the core of our KK relay system. But I can't say that KK relay is a totally efficient system because we employ symmetric relays (the same hand patterns follow the same sequences) in order to be easier to remember and bid within the allotted time. Because bridge is a timed event, information theory has to give way a bit to being able to bid timely.

I have always been fascinated by game theory. Dave and I corresponded with respect to several of the examples in the game theory section of the book. It is difficult for most bridge players to accept that a player (both as declarer and defender) might make different plays with exactly the same hands and for the same contract. Even if there may be limited usefulness of a mixed strategy in practice, the concept of "what is right" must include its consideration. By addressing the application of game theory strategy to bridge, Dave is advancing the ball in furthering our understanding of the game.

Long before computers were used for bridge, I was active in using them to better understand the game of backgammon. That backgammon is a game of perfect information makes it easier to analyze and "solve". Today there are backgammon computer programs that play better than the best human. In this regard, bridge is a much "larger" game and, regrettably, we have not made much progress. Think of this book as a "starter" which in time may become outdated as artificial intelligence continues to improve.

Bridge is a beautiful game and has been an important part of my life. In some senses it is like an infinitely large onion where you can peel off layer after layer, never able to get to the middle. "Bridge, The Cutting Edge" removes another layer exposing the shiny skin of the new layer below.

*(Kit Woolsey is a world class bridge and backgammon player and author of numerous books in both fields. A mathematician by training, he has been inducted into both the bridge and backgammon halls of fame. He wrote his own computer programs for backgammon analysis in the early 1980's and ran the first internet backgammon server. He is a leading expert on the strategy of using the backgammon doubling cube which employs "equity analysis" and extended concepts of "expected value." Kit's most recent bridge book, *KK Relay* written with Kate McCallum, uses relays which take advantage of the principles of information theory to optimize the available bidding space.)

# INFORMATION SCIENCE

In this section of the book, we are going to look at what is called "information science" or "information theory" as it may be applied to bridge bidding and defensive carding. Whether it is cellphones or satellites, the goal of information science is to bundle information in a way that is most efficient and can be transmitted with the least cost.

The concept of information entropy was introduced by Claude Shannon in his 1948 paper "A Mathematical Theory of Communication." "Shannon's entropy" is a measure of the potential reduction in uncertainty in the receiver's knowledge. The process of gaining information is equivalent to the process of losing uncertainty. Entropy is a measure of randomness. As you receive information, you increase your understanding of your subject and reduce its entropy.

For example, imagine you are standing behind a closed door. There are between 1 and 8 people in the room on the other side. You would like to know how many people are in the room, and you can ask only yes or no questions. What should your first question be? How about trying, "Are there more than four people?" The answer is "yes." Second question, "Are there more than 6 people?" The answer is "no." Third question, "Are there more than 5 people. The answer is "yes." You now know there are 6 people in the room. It took you three questions, which is not coincidentally $2 \times 2 \times 2 = 2^3$ or "two to the third power." In base 2, with 1 = yes and 0 = no, this stream of information would be written as "101." Each answer, which is a packet of information, is also referred to as a "bit".

Suppose you had asked on your first guess, "Are there more than 5 people?" Now, you cannot be assured of finding out how many people there are in 3 guesses. In response to a yes-no question, Shannon's entropy is most quickly reduced when the likelihood of each answer is 50%. Think of it this way: you could ask, “Is there one person?”, followed by “Are there two people?”, continuing until you get an answer of “yes.” This is not efficient in that it may take you 7 questions.

In bridge, one role of bidding and—to a lesser extent—defensive card play, is to reduce the entropy of each deal. We are going to look at ways of doing so. The most obvious example is the “invitation and acceptance.” Assume one partner opens 1NT, showing 15-17 high card points (HCP), and the other partner has a balanced hand with no four card or longer major. Assuming the partnership goal is to bid game with 25 HCP (yes, it can be more complicated depending on vulnerability, form of scoring, state of the match, the consequences of bidding 2NT and going down a trick, etc.), how many high card points does responder need to bid a natural and invitational 2NT? If you can only ask one question, information science tells us that we want to ask a yes-no question that has a 50% chance of being answered yes. That would suggest that responder should have 9 HCP. The problem is a bit more complicated because the probability of HCP distributions among 15-17 HCP hands is not uniform. The most likely high card point holding is 10 and it reduces as you go higher and lower from there. A first approximation is that, for a 1NT opener, 15 HCP occurs 43.7%, 16 HCP occurs 32.8% and 17 HCP occurs 23.4% of the time. Accordingly, if responder is going to invite with all 9 HCP hands, opener should accept with all 17 HCP hands and about the best 3/4ths of the 16 HCP hands (where “best” is based on a more sophisticated evaluation of how good or bad a hand is). This is a basic application of information science to bridge bidding.

Many experts advocate “heavy invites” and “light acceptances,” perhaps accepting with all 16 counts in the above example. Their rationale is that they want to avoid going minus in 2NT when opener has a real minimum. This is a valid consideration, but it is difficult to know when your invite puts a plus score in jeopardy. It makes the invite less efficient, and you have a tradeoff. It also may mean you will miss more games.

There is an element of the game once pointed out to me by American bridge pro Chris Compton. Defenders try to beat the contract. It is very common to see a push board at IMPs where both pairs go down a trick. One table was in 3NT, the other in 2NT. The defenders in 3NT are making sure they beat 3NT, caring not so much about beating it two tricks. The defenders in 2NT are taking risks to beat 2NT, caring not so much if declarer makes three. But when 3NT is bid and made, the game bonus makes up for some of those down ones and down twos. This is one aspect of the game where double dummy analysis falls short.

# *Bidding Space*

Bridge bidding is a "limited language" in that two partners attempt to use their available bids to convey information to one another in an effort to reach the optimal partnership contract. Generally, the more precisely each partner can describe his hand to the other, the more likely it is that the contract reached will be the right one. There are some exceptions to this proposition which we will explore, but I believe it to be a reasonable starting point.

It is not possible to have a bid to describe every hand. The number of possible 52-card bridge deals is:

*53,644,737,765,488,792,839,237,440,000*
*(~53 followed by 27 zeros).*

The number of possible auctions (with the opponents passing throughout) is:

*68,719,476,735 (a mere 68.7 billion, ~68 followed by 9 zeros).*

If you divide one by the other, you will see that for each bidding sequence, on average, there are approximately $7.8 \times 10^{17}$ possible hands.

Some bidding sequences are shared dialogs where each partner is describing elements of their hand to the other. Other bidding sequences are interrogations where one partner describes their hand to the other partner who is asking questions. And some bidding sequences flip back and forth where the auction may start as a dialog but at some point, one partner takes control.

Let's analyze what it is to have a simple interrogation bidding sequence. We are going to have one partner do the asking (the "captain"), and the other do the answering (the "crew"). At each turn, the captain is going to use just the next step in the sequence as the asking bid. We are going to look at how many pieces of information one bidding partner can convey to another (example: "I have two aces," "I have four spades," "I have 8 or more high card points.") We are going to refer to each piece of information as a "packet." We are also going to assume that the crew is going to answer correctly, that is to say there is no "mutiny."

Think in terms of how many available steps you have in bidding. Start with 1♣. If you can only bid as high as 1♢, you have one available step. If you can bid as high as 1♡, you have two available steps (1♢ and 1♡) and can convey two packets of information and if you can bid as high as 1♠ you can convey three packets. But it is when you get to use more than three steps that the bidding theory is more interesting.

Assume four available steps. The following auctions are available:
1♣-1♢
1♣-1♡
1♣-1♠
1♣-1NT

But starting with 1♣-1♢, you can ask again with 1♡ (next step relay) and get two packets:
1♣-1♢-1♡-1♠
1♣-1♢-1♡-1NT
So, in the aggregate, if you have four available steps, you can transmit five packets of information.

Let's go to five available steps (2♣):
1♣-1♢-1♡-1♠
1♣-1♢-1♡-1NT
1♣-1♢-1♡-2♣
1♣-1♡-1♠-1NT
1♣-1♡-1♠-2♣
1♣-1♠
1♣-1NT
1♣-2♣

With five steps, you can transmit eight packets. I will tell you that with six steps, you can transmit thirteen packets; with seven steps, you can transmit twenty-one; with eight steps, thirty-four.

Have you read the book or seen the movie *The Da Vinci Code*? If you did you might recognize this pattern of numbers. Leonardo of Pisa or Leonardo Fibonacci ("fi" meaning "child of" in Italian) was born around 1170 to Guglielmo Bonacci, a wealthy Italian merchant. He wrote a mathematics book that identified an important series which now bears his name.

The Fibonacci series are the numbers in the following integer sequence: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, … The first two numbers in the Fibonacci sequence are 0 and 1, and each subsequent number is the sum of the previous two.

As the sequence approaches infinity, the numbers converge in proportion to approximately 1.6180, often referred to as the "golden ratio." The Fibonacci series and the golden ratio can be found in nature, music, architecture and investment strategy as well as information theory. For example, the petals of a sunflower or the leaves of an artichoke outwardly spiral in accordance with the Fibonacci series. Many of Mozart's sonatas break at .618 of the way through and the motto (the "da-da-da-da") in Beethoven's Fifth can be found at the beginning, the end, and at measure 372 out of 601 which, as you may now guess, is .618 of the way through the symphony. Both Mozart and Beethoven had an interest in mathematics. And it has been claimed that the Parthenon in Greece and the Great Pyramids in Egypt both were constructed in proportion to the golden ratio, but those claims have been disputed and widely rejected.

So, what does this mean to bridge bidding? The more steps you have available, the more information you can convey and the amount of information you can convey (the number of packets) increases rapidly (by approximately 1.618 times the previous amount) as the number of steps gets greater.

How many steps are "available"? Assuming no adverse bidding, for each level there are five steps (♣, ◊, ♡, ♠ and NT.) I should add that some experts, notably in Australia and New Zealand, do take advantage of one additional step by playing what are called "forcing pass systems," where their big opening bid is an initial pass. This works well in giving them one more useful step but it does require them to use some other bid when they don't have any sort of opening bid at all and that creates some complexity and chaos. (That meaningless bid is often called a fertilizer bid or "fert" for short. You can figure out why.) Because of the fert, forcing pass systems are not permitted by the ACBL, but they do have theoretical merit and may be found in some international competition.

# *Game-Ordered Bidding*

A deck of cards has four suits and they come in a specific order for purposes of bidding, starting with clubs up through spades and then notrump. Call that "natural bidding." This order is not, however, the order in which the suits occur for purposes of scoring. Game scoring starts with 3NT, followed by 4♡/4♠, and then with 5♣/5♢. Call that "game-ordering."

Natural bidding means that in many auctions you have more steps, and therefore more packets of information, available in bidding to 5♢ than you do in bidding to 4♡. Ideally you would want to have as many steps available to bid to 4♡ as you do to bid to 4♠, 5♣ and 5♢ (Perhaps an argument could be made that because major suit games and slams score more, they deserve more bidding room and not less). I have some examples where reordering the meaning of bids can improve your bidding by attempting to equalize the number of steps for each of the possible suit contracts.

Compare the simple auctions of 1♡-3♡ and 1♣-3♣ (in both cases, assume that the raise is natural and invitational). If opener has a big hand and wants to investigate slam, they have 4 steps available between 3♡ and 4♡ and 9 steps between 3♣ and 5♣. This makes slam bidding in clubs more effective than slam bidding in hearts. One alternative is to play 1♡-2♠ as an invitational heart raise (and correspondingly playing 1♠-2NT as a spade raise), which gives the partnership additional bidding room, but it comes with a cost. If bidding was truly to be game-ordered, one would open 1♣ to show notrump (a balanced range), 1♢ to show hearts and 1♡ to show spades. We will get there later in the book, but some computers using artificial intelligence have, by bidding millions of hands without any imposed bidding agreements, converged on "transfer openings," which is simply game-ordered bidding.

Bidding slam starting with 2NT is notoriously difficult because you have already wasted 9 steps with your opening bid. A common treatment after bidding Stayman and a major suit response is to use the other major as an artificial slam try. If the auction starts 2NT-3♣-3♡, it is possible to use 3♠ as inviting slam in hearts. This bid is game-ordered because there is no other more important use for the 3♠ bid which is the next step. But look what happens when the auction starts 2N-3♣-3♠. Standard treatment is to use 4♡ as the spade slam try. You have wasted all the bidding room between 3♠ and 4♠ for this purpose. A game-ordered approach would be to use 4♣ as the artificial slam try, 4♢ to show clubs and 4♡ to show diamonds. This gives the partnership a couple of useful steps to show controls or degree of strength (e.g., last train).

A common problem is Blackwood. Those of you who have played for any length of time have run into the problem in bidding minor suit slams of not having enough room between 4N and 5♣ or 5♢. Those of you who play keycard Blackwood, where you show the trump king as a fifth ace and differentiate between holding and not holding the trump queen have also had the problem when trump is hearts. “1430 responses” are an attempt to reduce the problem of lacking enough steps when hearts are trump, but, perhaps, the better answer is to use some form of “kickback” where 4♢ is Blackwood for clubs, 4♡ is Blackwood for diamonds, 4♠ is Blackwood for hearts and 4NT is Blackwood for spades. Note that this preserves the same number of steps (4) for each ask and answer.

Here is an example that is a bit more exotic. There is a convention credited to Eric Kokish to show really big balanced hands by opening 2♣, and when partner bids 2♢, rebidding 2♡ to show either hearts or a game forcing balanced hand. Responder is expected to rebid 2♠, and now opener’s bid of 2NT is forcing. It is a good convention, many experts play some variation of it, and I recommend it to you. But the problem becomes how do you then show hearts? In its simplest form, opener is forced to bid 2♣-2♢-3♡. This has a couple of problems. It is inefficient in that you have used up all the steps between 2♢ and 3♡. It also creates a problem if you have a really good hand that isn't quite game forcing. So, the first fix was to swap the 3♣ and 3♡ rebids. If opener had just hearts, he could bid 2♣-2♢-2♡-2♠-3♣ showing hearts, and now responder has 3♢ available as a second negative and the partnership could stop in 3♡. With hearts and clubs and a game forcing hand, opener uses 3♡ as a "replacement bid" showing hearts and clubs. A newer improvement, that is sometimes referred to as "tumbling suits," is to take that reordering one step further and use game-ordered bidding so that 3♣ is hearts, 3♢ is hearts and spades, 3♡ is hearts and clubs, and 3♠ is hearts and diamonds. This preserves the same number of steps between the second suit shown and its corresponding game bid.

Similar to the Kokish example is playing new suits as nonforcing as responder to opener's bid of a "weak two." As such preempts become less disciplined, there is more to be gained (and less lost) by allowing responder to bid a new suit without creating a force. When opener starts with 2♡, responder can bid 3♠ as game forcing with spades. But when opener starts with 2♠, it would be useful to be able to show hearts at the three level, both forcing and nonforcing. One solution, is to play 2♠-3♣ as hearts and 2♠-3♡ as clubs. This gives opener a rebid over 3♣ to show their fit for hearts.

It is a difficult problem when the opponents preempt at the three level and partner overcalls 3NT. Partner's bid is not well-defined, it may be a single suit with a stopper, a strong balanced hand, or a really strong balanced hand not suitable for doubling. It is not a frequent occurrence and I would hazard a guess that most partnerships do not have many special agreements. Our suggestion is game-ordered bidding by advancer where a 4♣ bid is the equivalent of having bid 4♡, a bid of 4♢ is the equivalent of 4♠, 4♡ is clubs and 4♠ is diamonds.

Look what happens. Take the auction (3♠)-3NT-(P)-? If 4♣ shows hearts, overcaller can take the transfer and bid 4♡ with a normal balanced hand, bid 4NT with a hand not interested in hearts (typically a single suited hand) and bid the middle step with slam aspirations. If the auction proceeds (3♠)-3NT-(P)-4◊, that is a 4♠ bid which we play as two suited with hearts (Michaels) and, again, overcaller has options available to further slam exploration. Similarly, 4♡ shows clubs and 4♠ shows diamonds. All of this is possible because of the use of game-ordering.

It has become increasingly popular to play 1M-2♣ as game forcing but not promising length in clubs. Using game-ordered rebids, 1M-2♣-2◊ showing hearts, 1M-2♣-2♡ showing spades and 1M-2♣-2♠ showing diamonds, gives the partnership the ability to agree on a major suit fit at the two level, preserving 5 steps of bidding room.

One more example (although I am sure you can find others based on your bidding system and existing partnership agreements): my wife, Anne, and I play a form of Precision known as "Meckwell Lite" where 1♣-1♡ is 8-11 HCP without 5 spades. While it would probably be advantageous to tumble our suits after a 1♡ bid, it would be a significant change that is more than we are prepared to make at this time (and may find me sleeping on the couch). This makes the auction 1♣-1♡-2♡ natural and forcing, but it does burn a lot of steps. To remedy that we play 1♣-1♡-2♡-2NT as a three card raise with opener's rebid over 2NT game-ordered in the same way as it is in "modified Kokish" (3♣ is hearts, 3◊ is hearts and spades, etc.). This regains some of what was lost.

GENERAL INTEREST

If you are a student of game theory, statistics, and math, *The Cutting Edge* is a great read. Dave applies the nuances of these strategies to the game we all love. The text is highly analytical.

*Ralph Katz, Bermuda Bowl Champion*

I enjoyed this a lot. I expect something will be new to most readers, and it's a nice collection of related ideas in one place.

*Franco Baseggio, hedge fund statistical arbitrage specialist*

[The book] presents bridge situations in a way that bridge players would understand them. It is much more important to be interested in thinking about things mathematically than to have previous knowledge.

*Greg Lawler, mathematics professor, University of Chicago*

I enjoyed the hands and analysis. Following the equilibrium mixed strategy is safe in a sense, but in practice you're only exploitable if the opponent knows you deviate and in which direction.

*Jonathan Weinstein, economics professor,*
*University of Washington in St. Louis*

**Dave Caprera** is a retired attorney from Denver, Colorado. He started playing bridge in college 50 years ago. His previous book for Master Point Press was Sleeping on the Couch. In addition to playing and writing, Dave is active as a coach in the USBF Junior Training Program.

Profits from the sale of this book will be donated to the USBF Junior Program

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